AKSHEYAA COLLEGE OF ENGINEERING

UNIT 3 –FREE VIBRATION

1. The following data relate to a shaft held in long bearings. Length of shaft = 1.2 m; Diameter of shaft = 14 mm; Mass of a rotor at midpoint =16 kg; Eccentricity of centre of mass of rotor from centre of rotor = 0.4 mm; Modulus of elasticity of shaft material=200 GN/m^2; Permissible stress in shaft material = 70 x 10^6 N/m^2. Determine the critical speed of the shaft and the range of speed over which it is unsafe to run the shaft. Assume the shaft to be massless. **May 2003**
2. In a single degree of damped vibrating system, a suspended mass of 3.75 kg makes 12 oscillations in 7 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.33 of its initial value in four oscillations. Determine: (i) stiffness of the spring, (ii) logarithmic decrement, (iii) damping factor and (iv) damping coefficient. **Dec 2003**
3. The barrel of a large gun recoils against a spring on firing. At the end of the firing, a dash pot is engaged that allows the barrel to return to its original position in minimum time without oscillation. Gun barrel mass is 400 kg and initial velocity of recoil is 20 m/s. The barrel recoils 1m. Determine spring stiffness and critical damping coefficient of dashpot. **Dec 2003**
4. A machine weighing 75 kg is mounted on springs and is fitted with dashpot to damp vibrations. There are three springs each of stiffness 10 kg/cm and it is found that the amplitude of vibration diminishes from 3.84 cm to 0.64 cm in two consecutive oscillations. Assuming damping force varies as the velocity, determine the resistance of the dashpot at unit velocity and compare the frequency of damped vibration with the frequency when the dashpot is not in operation. **May 2004**
5. The two equal masses of 500 kg and radius of gyration 37.5 cm are keyed to opposite ends of a shaft 60 cm long. The shaft is 7.5 cm dia. for the first 25 cm and 12.5 cm dia. for the next 10 cm and 8.75 cm dia. for the remainder of its length. Find the natural frequency of the free torsional vibration of the system and the position of the node. Take G = 8 x 10^6 N/cm^2. **May 2004**
6. The flywheel of an engine driving a dynamometer has a mass of 180 kg and a radius of gyration of 30 mm. The shaft at the flywheel and has an effective length of 250 mm and is 50 mm diameter. The armature mass is 120 kg and its radius of gyration is 22.5 mm. The dynamo shaft is 43mm diameter and 200 mm effective length. Calculate the position of node and frequency of torsional oscillation. Take C : 83 kN/mm^2. **Dec 2004**
7. The two rotors A and B are attached to the end of a shaft 500 mm long. The mass of the rotor A is 300 kg and its radius of gyration is 300 mm. The corresponding values of rotor B are 500 kg and 450 mm respectively. The shaft is 70 mm in diameter for first 250 mm; 120 mm diameter for next 70 mm and 100 mm diameter for the remaining length. If G = 80 GN/m^2, find (i) the position of the node and (ii) the frequency of torsional vibration. **May 2005**
8. An instrument vibrates with a frequency of 1 Hz when there is no damping. When damping is provided, the frequency was observed to be 0.9 Hz. Find the (i) damping factor, and (ii) logarithmic decrement. **May 2005**
9. A shaft of 180 mm diameter is supported by two bearings 2.5 meters apart. It carries three discs of mass 250 kg, 500 kg and 200 kg at 0.6 m, 1.5 m and 2 m from the left hand. Assuming mass of the shaft as 190 kg/m, determine critical speed of the shaft. Take E=211GN/m^2. **Dec 2005**
10. A vibrating system consists of a mass of 1 kg, spring of stiffness 5.6 N/mm and a dash pot of damping coefficient of 40 N/m/sec. Find: (i) damping factor, (ii) logarithmic decrement, and (iii) ratio of two consecutive amplitudes. **Dec 2005**

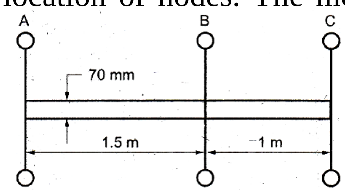
11. In a single-degree damped vibrating system, a suspended mass of 8 kg makes 30 oscillations in 18 seconds, The amplitude decreases to 0.25 of the initial value after 5 oscillations. Determine : (i) the spring stiffness; (ii) logarithmic decrement; (iii) damping factor; and (iv) damping coefficient. **May 2006**
12. Derive the expression for the natural frequency of free transverse vibrations for a shaft subjected to a number of point loads. **Dec 2006**
13. A vibrating system consists of mass of 8 kg, spring of 5.6 N/mm stiffness and a dashpot of damping coefficient of 40 N/m/s. Find damping factor, logarithmic decrement and ratio of two consecutive amplitudes. **Dec 2006**
14. A vertical steel shaft of 20 mm diameter is mounted in long bearings which are 1.2 m apart and carries 150 N of disc at its middle. The eccentricity of the centre of gravity of the disc from the centre of the rotor is 0.3 mm. Taking Young's modulus as 200 GN/m^2 and permissible stress as 74 MN/m^2, calculate the critical speed of the shaft and range of safe speed. **Dec 2006**
15. The flywheel of an engine, that drives a dynamo, has a weight of 2000 N and radius of gyration of 30 mm. The shaft at the flywheel end has an effective length of 275 mm and is 55 mm diameter. The armature mass is found to be 125 kg with radius of gyration of 25 mm. The dynamo shaft is 45 mm diameter and 200 mm effective length. Calculate the position of node and frequency of torsional oscillation. Take C =83 kN/mm^2. **Dec 2006**
16. A stepped shaft is 0.05 m in diameter for the first 0.6 m length, 0.08 m diameter for the next 1.8 m and 0.03 m diameter for the remaining 0.25 m length. While the 0.05 m diameter end is fixed, the 0.03 m diameter end of the shaft carries a rotor of mass moment of inertia 14.7 kg-m^2. If the modulus of rigidity of the shaft material is 0.83 x $l0^{11}$ N/m^2, find the natural frequency of torsional oscillations, neglecting the inertia effect of the shaft. **May 2007**
17. Determine the torsional frequency of a disc mass 96 kg, which is on the vertical rod of dia 40 mm. The rod is 1.8 m in length and the disc is fixed at 1 m from the top end. Both ends of the rods are fixed. The radius of gyration of disc is 0.4 m. Take G=85 GPa. **Dec 2007**
18. A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m^2. Determine the frequency of longitudinal and transverse vibrations of the shaft. **Dec 2007**
19. A steel shaft 1.5 m long is 95 mm in diameter for the first 0.6 m of its length, 60 mm in diameter for the next 0.5 m of the length and 50 mm in diameter for the remaining 0.4 m of its length. The shaft carries two flywheels at two ends, the first having a mass of 900 kg and 0.85 m radius of gyration located at the 95 mm diameter end and the second having mass of 700 kg and 0.55 m radius of gyration located at the other end. Determine the location of the node and the natural frequency of free torsional vibration of the system. The modulus of rigidity of shaft material may be taken as 80 GN/m^2. **Dec 2007**
20. A body of mass of 50 kg is supported by an elastic structure of stiffness l0 kN/m. The motion of the body is controlled by a dashpot such that the amplitude of vibration decreases to one-tenth of its original value after two complete cycles of vibration. Determine (i) the damping force at 1 m/s; (ii) the damping ratio, and (iii) the natural frequency of vibration. **May 2008**
21. Two parallel shafts A and B of diameters 50 mm and 70 mm respectively are connected by a pair of gear wheels, the speed of A being 4 times that of B. The mass moment of inertia of the flywheel is 3 kg-m^2 is mounted on shaft A at a distance of 0.9 m from the gears. The shaft B also carries a flywheel of mass moment of inertia 16 kg-m^2 at a distance of 0.6 m from the gears. Neglecting the

effect of the shaft and gear masses calculate the fundamental frequency of free torsional oscillations and the positions of node. Assume modulus of rigidity as 84 $GN/m^{2.}$ **May 2008**

22. A mass of 7.5 kg, hangs from a spring and makes damped oscillations. The time for 60 oscillations is 35 seconds and the ratio of the first and seventh displacement is 2.5. Find (i) the stiffness of the spring and (ii) the damping resistance in N/m/s. If the oscillations are critically damped, what is the damping resistance required in N/m/s? **(Nov 2008)**
23. The flywheel of an engine driving a dynamo has a mass of 180 kg and a radius of gyration of 30 mm. The shaft at the flywheel end has an effective length of 250 mm and is 50 mm diameter. The armature mass is 120 kg and its radius of gyration is 22.5 mm. The dynamo shaft is 50 mm diameter and 200 mm effective length. Calculate the position of node and frequency of torsional oscillation. C = 83 kN/mm^2. **(Nov 2008)**
24. (a) (i) Derive an expression for the natural frequency of single degrees of freedom system.
    (ii) Calculate the whirling speed of a shaft 25 mm diameter and 0.7 m long carrying a mass of 1 kg at its midpoint. The density of the shaft material is 40000 kg/m^3, and E=210 GN/m^2. Assume the shaft to be freely supported. **May 2009**
25. The moment of inertia of three rotors A, B and C are respectively 0.3, 0.6 and 0.l8 $kg.m^2$.The distance between A and B is 1.5 m and B and C is 1 m. The shaft is 70 mm in diameter and the modules of rigidity for the shaft material is 84 x $l0^9$ N/m^2. Find: (i) the frequencies of torsional vibrations; (ii) position of nodes; and (iii) amplitude of vibrations. **May 2009.**
26. A shaft is simply supported at its ends and is 40 mm in diameter and 2.5 m in length. The shaft carries three point loads of masses 30 kg, 70 kg, and 45 kg, at 0.5 m, 1m, 1.7 m respectively from the left support. The weight of the shaft per meter length is given as 73.575 N. The Young's modulus for the material of the shaft is 200 GN/m^2. Find the critical speed of the shaft. **Nov 2009.**
27. A machine weighs 5 kg and is supported on spring and dashpots. The total stiffness of the springs is l2 N/mm and damping coefficient is 0.2 N/mm/s. The system is imparted to the mass. Determine: (i) the displacement and velocity of mass as a function of time; and (ii, the displacement and velocity after 0.4 s. **Nov 2009.**
28. A coil of spring stiffness 4N/mm supports vertically a mass of 20 kg at the free end. The motion is restricted by oil dashpot. It is found that the amplitude at the beginning of the fourth cycle is 0.8 times the amplitude of the previous vibration. Determine the damping force per unit velocity. Also find the ratio of the frequency of damped and undamped vibrations. **May 2010**
29. The two rotors A and B are attached to the end of a shaft 500 mm long. The mass of the rotor A is 300 kg and its radius of gyration is 300 mm. The corresponding values of the rotor B are 500 kg and 450 mm respectively. The shaft is 70 mm in diameter for the first 250 mm; 120 mm for the next 70 mm and 100 mm diameter for the remaining length. The modulus of rigidity for the shaft materials is 80 GN/m^2. Find: (i) the Position of the node, and (ii) the frequency of torsional vibration. **May 2010**
30. Determine: (i) the critical damping coefficient, (ii) the damping factor, (iii) the natural frequency of damped vibrations, (iv) the logarithmic decrement and (v) the ratio of two consecutive amplitudes of a vibrating system which consists of a mass of 25 kg, a spring of stiffness 15 kN/m and a clamper. The clamping provided is only l5% of the critical value. **Nov. 2010.**
31. A shaft of length 1.25 m is 75 mm in diameter for the first 275 mm of length, 125 mm in diameter for the next 500 mm length, 87.5 mm in diameter for the next 375 mm length and 175 mm in diameter for the remaining 100 mm of its length. The shaft carries two rotors at two ends. The mass

moment of inertia of the first rotor is 75 kg-m^2 whereas of the second rotor is 5.0 kg-m^2. Find the frequency of natural torsional vibrations of the system. The modulus of the .rigidity of the shaft material may be taken as 80 GN/m^2.**Nov. 2010**

32. A machine of mass 75 kg is mounted on springs and is fitted with a dashpot to damp out vibrations. There are three springs each of stiffness 10 N/mm and it is found that the amplitude of vibration diminishes from 38 mm to 6 mm in two complete oscillations. Assuming that the damping force varies as the velocity, determine: (i) the resistance of the dashpot at unit velocity, (ii) the ratio of the frequency of the damped vibration to the frequency of the undamped vibration, (iii) the periodic time of the damped vibration. **Nov. 2010**
33. A shaft 1.5 m long supported in flexible bearings at the ends carries two wheels each of 50 kg mass. One wheel is situated at the centre of the shaft and the other at a distance of 375 mm from the centre towards left. The shaft is hollow of external diameter 75 mm and internal diameter 40 mm. The density of the shaft material is 7700 kg/m^3 and its modulus of elasticity is 200 GN/m^2. Find the transverse natural frequency of the shaft, taking into account the mass of the shaft. **Nov. 2010**
34. A single cylinder oil engine drives directly a centrifugal pump. The rotating mass of the engine, flywheel and the pump with the shaft is equivalent to a three rotor system as shown in figure. The mass moment of inertia of the rotors A, B and C are 0.15, 0.3 and 0.09 kg/m^2. Find the natural frequency of the torsional vibration and location of nodes. The modulus of rigidity for the shaft material is 84 kN/m^2. **Nov. 2010**



35. A shaft 50 mm diameter and 3 meters long is simply supported at the ends and carries three point loads of 1000N, 1500N and 750 N at l m, 2 m and 2.5m respectively from the left support. The Young's modulus for the shaft material is 200 GN/m^2. Find the frequency of transverse vibration **Nov 2011** (8).
36. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. Take E = 200 GN/m^2. **Nov 2011** (8).
37. An instrument vibrates with a frequency of l Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 0 9 Hz. Find the damping factor and logarithmic decrement. **Nov 2011** (6)
38. A 4-cylinder engine and fly wheel coupled to a propeller are approximated to a 3 rotor system in which the engine is equivalent to a rotor of moment of inertia 800 kg-m^2 wheel to a second rotor of 320 kg-m^2 and the propeller to a third rotor of 20 kg-m^2. The first and second rotors being connected by 50 mm diameter and 2 metre long shaft and the second and third rotor being connected by a 25 mm diameter and 2 metre long shaft. Neglecting, the inertia of the shaft and taking its modulus of rigidity as 80 GN/m^2, determine the natural frequencies of torsional oscillations and the position of the nodes. **Nov 2011** (10).
39. A 5 kg mass attached to the lower end of a spring, whose upper end is fixed, vibrates with a natural period of 0.45 sec. Determine the natural period when 2.5 kg mass is attached to the midpoint of the same spring with the upper and lower ends fixed. **May 2012** (6)

40. A vibrating system is defined by the following parameters: m=3 kg, k = 100 N/m, C = 3 N-s/m. Determine the damping factor, the natural frequency of damped vibration, logarithmic decrement, the ratio of two consecutive amplitudes and the number of cycles after which the original amplitude is reduced to 20 percent. **May 2012(**10)
41. A single rotor system has a natural frequency of 5 Hz, what length of steel rod of diameter 20 mm should be used for this rod? The inertia of the mass fixed at the free end is 0.0098 kg-m^2. Take G=0.85 x $l0^{11}$ N/m^2. **May 2012**
42. The mass of a single degree damped vibrating, system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine: i) Stiffness of the spring, (ii) Logarithmic decrement, and (iii) Damping factor. **Nov 2012**
43. Derive an expression for the frequency of free torsional vibrations for a shaft fixed at one end and carrying a load on the free end. **Nov 2012** (8)
44. What is meant by torsionally equivalent length of a shaft as referred to a stepped shaft? Derive the expression for the equivalent length of a shaft which has several steps. **Nov 2012** (8)
45. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E=200 GN/m^2. **May 2013**
46. A machine of mass 75 kg is mounted on springs and is fitted with a dashpot to damp out vibrations. There are three springs each of stiffness 10 N/mm and it is found that the amplitude of vibration diminishes from 38.4 mm to 6.4 mm in two complete oscillations. Assuming that the damping force varies as the velocity, determine; (i) the resistance of the dashpot at unit velocity; (ii), the ratio of the frequency of the damped vibration to the frequency of the undamped vibration; and (iii) the periodic time of the damped vibration. **May 2013.**
47. A machine weighs 18 kg and is supported on springs and dashpots. The total stiffness of the springs is 12 N/mm and damping is 0.2 N/mm/s. The system is initially at rest and a velocity of 120 mm/s is imparted to the mass. Determine: (1) the displacement and velocity of mass as a function of time, and (2) the displacement and velocity after 0.4 s. **Nov 2013**
48. A torsional system is shown in figure below, find vibrations and the positions of the nodes also find G = 84 x $l0^9$ N/m^2. **Nov 2013**