AKSHEYAA COLLEGE OF ENGINEERING

Puludivakkam, (Near Nelvoy Junction), Maduranthagam Taluk,
Kancheepuram 603 314, Tamil Nadu, India.
Phone: + 91 44 27568004-02-05, email: principal@ace.ac.in
(An ISO 9001:2008 Certified Institution)

Department of Science & Humanities

Subject Name: MATHEMATICS -II

Subject Code: MA6251 **I Year/II Semester (2015-16)**

PART-A

I.VECTOR CALCULUS

1. Find 'a' , such that $(3x-2y+z)\vec{i}+(4x+ay-z)\vec{j}+(x-y+2z)\vec{k}$ is solenoidal.
2. Show that, $\vec{F}=yz\vec{i}+zx\vec{j}+xy\vec{k}$ is irrotational.
3. Find the values of a,b,c so that the vector $\vec{F}=(x+y+az)\vec{\imath}+(bx+2y-z)\vec{\jmath}+(-x+cy+2z)\vec{k}$ may be irrotational.
4. Define solenoidal vector function. If $\vec{V}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+2\lambda z)\vec{k}$ is solenoidal, find the value of λ.
5. Prove that $Culr(grad\ \phi)=\vec{0}$
6. State Gauss divergence theorem.
7. State Stoke's theorem.
8. State Green's theorem, in a plane.
9. Find Φ if $\nabla\Phi=(6xy+z^3)\vec{\imath}+(3x^2-z)\vec{\jmath}+(3xz^2-y)\vec{k}$.
10. Find the unit normal vector to the surface $x^2+xy+z^2=4$ at (1,-1,2)

II.ORDINARY DIFFERENTIAL EQUATIONS

11. Solve the equation $(D^2-6D+13)y=0$
12. Find the particular integral of $(D^2-2D+1)y=\cosh x$.
13. Find the particular integral of $(D^2+2D+1)y=e^{-x}cosx$
14. Find the particular integral of $(D^2-4D+4)y=x^2e^{2x}$.
15. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=0$.
16. Find the Wronskian of y_1,y_2 of $y''-2y'+y=e^x\log x$.
17. Solve the equation $x^2y''+xy'+y=0$.
18. Convert $(3x^2D^2+5xD+7)y=\frac{2}{x\log x}$ into an equation with constant coefficients.
19. Transform the equation into a linear differential equation with constant coefficient.
20. Transform the equation $(2x+3)^2\frac{d^2y}{dx^2}-2(2x+3)\frac{dy}{dx}-12y=6x$ into a differential equation with constant coefficients.

III. LAPLACE TRANSFORM

21. Is the linearity property applicable to $L\left[\frac{1-\cos t}{t}\right]$?
22. Find the Laplace transform of $\frac{t}{e^t}$
23. Find the Laplace transform of $f(t)=\frac{1-e^{-t}}{t}$.
24. Find Laplace transform of $t\sin 2t$.

25. Define periodic function with an example.
26. Stat the first shifting theorem on Laplace transforms.
27. Evaluate $L^{-1}\left[\frac{1}{s^2+6s+13}\right]$
28. Find $f(t)$ if the Laplace transform of $f(t)$ is $\frac{s}{(s+1)^2}$.
29. Find $L^{-1}\left[\frac{s}{(s+2)^2+1}\right]$.
30. Verify initial value theorem for the function $f(t)=ae^{-bt}$.

IV. ANALYTIC FUNCTIONS

31. State the Cauchy-Riemann equations in polar coordinates satisfied by an analytic function.
32. Show that the function $f(z)=\bar{z}$ is nowhere differentiable.
33. Prove that every analytic function $w=u(x,y)+iv(x,y)$ can be expressed as a function of z alone.
34. Find the constants a,b,c if $f(z)=x+ay+i(bx+cy)$ is analytic.
35. Prove that $w=z^2$ is analytic and hence , find $\frac{dw}{dz}$.
36. Show that the function $v = e^x \sin y$ is harmonic.
37. Verify whether the function $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic.
38. Find the invariant points of the transformation $w=\frac{2z+6}{z+7}$.
39. Prove that a bilinear transformation has atmost two fixed points.
40. Find the map of the circle $|z|=3$ under the transformation $w=2z$.

V.COMPLEX INTEGRATION

41. State Cauchy integral formula.
42. What is the value of the integral $\int_C \frac{3z^2+7z+1}{z+1}dz$ where C is $|z|=\frac{1}{2}$?
43. Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)}$, Where C is the circle $|z|=\frac{1}{2}$.
44. Evaluate $\int_C \frac{z+4}{z^2+2z}dz$ Where C is the circle $\left|z-\frac{1}{2}\right|=\frac{1}{3}$.
45. Evaluate $\int_C \frac{z}{z-2}dz$ where C is $a)|z|=1,\ b)|z|=3$.
46. Calculate the residue of $f(z)=\frac{e^{2z}}{(z+1)^2}$ at its pole.
47. Find the residue of the function $f(z)=\frac{4}{z^3(z-2)}$ at a simple pole.
48. If $f(z)=\frac{-1}{z-1}-2\left(1+(z-1)+(z-1)^2+.....\right)$, find the residue of $f(z)$ at $z=1$.
49. Expand $f(z)=\sin z$ in a Taylor series about origin.
50. State Cauchy's residue theorem.

PART-B

I.VECTOR CALCULUS

1. Find a and b so that the surface $ax^3 - by^2z - (a+3)x^2 = 0$ and $4x^2y - z^3 - 11 = 0$ cut orthogonally at the point $(2,-1,-3)$.
2. Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. .
3. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} - z\vec{k}$ from t=0 to t=1 along the curve $x = 2t^2, y = t, z = 4t^3$.
4. Verify Green's theorem in the XY plane for $\int_C \left[(3x - 8y^2)dx + (4y - 6xy)dy\right]$ where C is the boundary of the region given by $x = 0, y = 0, x + y = 1$.
5. Verify Green's theorem for $\vec{V} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0\, and\; y = b$.
6. Verify the divergence theorem for the function $\vec{A} = x^2\vec{i} + z\vec{j} + yz\vec{k}$ over the cube $x = \pm 1, y = \pm 1, z = \pm 1$
7. Verify Gauss-divergence theorem for the vector function $\vec{F} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + 2\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ *and* $x = a, y = a, z = a$
8. Verify Stoke's theorm for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ taken around the rectangle formed by the lines $x = 0, x = a, y = o, y = b$.
9. Using Stoke's theorem , evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = y^2\vec{i} + x^2\vec{j} - (x+z)\vec{k}$ and C is the boundary of the triangle with vertices at $(0,0,0),(1,0,0)\, and\; (1,1,0)$.
10. Verify Gauss-divergence theorem for the vector function $\vec{F} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + 2\vec{k}$ over the cube bounded by $x = 0, y = 0, z = 0$ *and* $x = a, y = a, z = a$.

II. ORDINARY DIFFERENTIAL EQUATIONS

11. solve $(D^2 + 4)y = x^2 \cos 2x$.
12. solve $(D^2 - 3D + 2)y = 2\cos(2x+3) + 2e^x$
13. Solve by the method of variation of parameters $2\frac{d^2y}{dx^2} + 8y = \tan 2x$.
14. Apply the method of variation of parameters to solve $(D^2+4)y= \sec 2x$.
15. Slove $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$
16. Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + y = \frac{12\log x}{x^2}$
17. Solve: $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$
18. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$
19. Solve $\frac{dx}{dt} + 4x + 3y = t, \frac{dy}{dt} + 2x + 5y = e^t$.
20. Solve $\frac{dx}{dt} + 2y = \sin 2t, \frac{dy}{dt} - 2x = \cos 2t$. Given x=1 and y=0 at t=0

III.LAPLACE TRANSFORM

21. Find $L\left[\frac{\cos at-\cos bt}{t}\right]$

22. Find the Laplace transform of $f(t)=te^{-3t}\cos 2t$.

23. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$.

24. Find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+4)}$.

25. Find the Laplace transform of $f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$ with $f(t+2a)=f(t)$

26. Find the Laplace transform of square wave function defined by $f(t)=\begin{cases} 1, & 0<t<a \\ -1, & a<t<2a \end{cases}$ and $f(t+2a)=f(t)$ for all t.

27. . Find the Laplace transform of the Half wave rectifier $f(t)=\begin{cases} \sin wt, & 0<t<\pi/w \\ 0 & \pi/w<t<2\pi/w \end{cases}$ and $f\left(t+\frac{2\pi}{w}\right)=f(t)$ for all t.

28. Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

29. Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

30. Solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=\sin t$, if $\frac{dy}{dt}=0$ and $y=2$ when $t=0$ using Laplace transform.

IV.ANALYTIC FUNCTIONS

31. Find the regular function whose imaginary part is $e^{-x}(x\cos y+y\sin y)$

32. Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y-\cos 2x}$

33. Prove that the real and imaginary parts of an analytic function are harmonic functions.

34. If $f(z)=u+iv$ is a regular function of z in a domain D then $\nabla^2(u^p)=p(p-1)u^{p-1}|f'(z)|^2$

35. Verify that the family of curves $u=c_1$ and $v=c_2$ cut orthogonally when $u+iv=z^3$.

36. Find the image of the circle $|z+1|=1$ in the complex plane under the mapping $w=\frac{1}{z}$

37. Show that the transformation $w=\frac{1}{z}$ transforms all circles and straight lines in the z-plane into circles or straight lines in the w-plane.

38. Find the image of the hyperbola $x^2-y^2=1$ under the transformation $w=\frac{1}{z}$.

39. Find the bilinear transformation which maps the points $0, 1, \infty$ into $i, 1, -i$.

40. Find the bilinear transformation which maps the points $1, i, -1$ onto $0, 1, \infty$, Show that the transformation maps the interior of the unit circle of the z- plane onto the upper half of the w plane.

V.COMPLEX INTEGRATION

41. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$ where C is $|z| = 3$.

42. Evaluate $\int_C \frac{z}{(z-1)(z-2)^2} dz$ where C is the circle $|z-2| = \frac{1}{2}$ by Cauchy's integral formula.

43. Evaluate $\int_C \frac{(z+1)}{(z^2+2z+4)^2} dz$ where C is the circle $|z+1+i| = 2$ by Cauchy's integral formula..

Expand the function $f(z) = \frac{z^2-1}{z^2+5z+6}$ in Laurent's series for $|z| > 3$ and $2 < |z| > 3$.

44. . Find the Laurent's series of $f(z) = \frac{7z-2}{z(z+1)(z+2)}$ in $1 < |z+1| < 3$.

45. Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$, where C is the circle $|z-i| = 2$ using Cauchy's residue theorem.

46. . Using Cauchy's residue theorem, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)^2} dz$ where C is the circle $|z| = \frac{3}{2}$.

47. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using contour integration.

48. Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+4)} dx$ using contour integration.

49. Evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$ using contour integration.

50. Evaluate $\int_0^{\infty} \frac{\cos mx}{x^2+a^2} dx$ using contour integration.

ALL THE BEST…….