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UNIT-I
PARTIAL DIFFERNTIAL EQUATIONS

PART-B

1. Form the PDE by eliminating the arbitrary function from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.(2015)
2. Form the PDE by eliminating the arbitrary function from $\phi\left(x^2+y^2+z^2, ax+by+cz\right)=0$. (2010,2012)
3. Form the PDE by eliminating arbitrary functions f and ϕ from $z = f(x+ct) + \phi(x-ct)$. (2011)
4. Form the PDE by eliminating the arbitrary function 'f' and 'g' from $z = x^2 f(y) + y^2 g(x)$.(2013)
5. Solve $z = px + qy + p^2q^2$. (2009,2015)
6. Find the singular integral if $z = px + qy + \sqrt{1+p^2+q^2}$.(2011,2013)
7. Find the singular integral of $z = px + qy + p^2 + pq + q^2$ (2012,2013)
8. Find the singular solution of $z = px + qy + p^2 - q^2$ (2014)
9. Solve $(y - xz)p + (yz - x)q = (x+y)(x-y)$. (2009)
10. Solve the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. (2010,2012)
11. Solve the partial differential equation $(mz - ny)p + (nx - lz)q = ly - mx$. (2011)
12. Solve the PDE $x(y-z)p + y(z-x)q = z(x-y)$. (2012,2014)
13. Solve the PDE $(x-2z)p + (2z-y)q = y - x$. (2012)
14. Solve $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$ (or) $x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2)$. (2013,2014)
15. Solve: $(y^2+z^2)p - xyq + xz = 0$ (2013)
16. Solve the Lagrange's equation $(x+2z)p + (2xz - y)q = x^2 + y$. (2014)
17. Solve $(x^2 - yz)p + (y^2 - xz)q = z^2 - xy$ (2015)
18. Solve $(D^2 + 2DD' + D'^2)z = \sinh(x+y) + e^{x+2y}$. (2009)
19. Solve the equation $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = \cos(2x+y)$ (2010,2012)
20. Solve $(D^2 - D'^2)z = \sin(2x+3y)e^{x-y}$.(2011)
21. Solve $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$. (2011)
22. Solve the equation $(D^2 + 3DD' - 4D'^2)z = \cos(2x+y) + xy$ (2012)
23. Solve $(D^2 + DD' - 6D'^2)z = y\cos x$ (2013,2014)
24. Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(2x+y)$ (2013,2014)
25. Solve $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{2x+4y}$ (2013)
26. Solve $(D^2 + 3DD' + 2D'^2)z = \sin(x+5y)$. (2014)
27. Solve $(D^2 - 2DD')z = x^3y + e^{2x-y}$ (2014)

28. Solve $\left(D^2-3DD'+2D'^2\right)z=(2+4x)+e^{x+2y}$ (2015)
29. Solve $\left(D^2-D'^2-3D+3D'\right)z=xy+7$. (2009)
30. Solve $\left(2D^2-DD'-D'^2+6D+3D'\right)z=xe^y$. (2010,2012)
31. Solve $\left(D^2-3DD'+2D'^2+2D-2D'\right)z=x+y+\sin(2x+y)$. (2011)
32. Solve $\left(D^2-2DD'+D'^2-3D+3D'+2\right)z=e^{2x-y}$. (2011)
33. Solve $\left(D^2-DD'+2D\right)z=e^{2x+y}+4$. (2012)

TWO MARKS

1. Form the partial differential equation by eliminating the constants a and b from $z=\left(x^2+a^2\right)\left(y^2+b^2\right)$.
2. Find the PDE of the family of spheres having their centers on the z-axis.
3. Form the PDE from $(x-a)^2+(y-b)^2+z^2=r^2$.
4. Form the PDE by eliminating the arbitrary constants a and b from $\log(az-1)=x+ay+b$.
5. Form the partial differential equation by eliminating the constants a and b from $z=\left(x^2+a\right)\left(y^2+b\right)$
6. Form a PDE by eliminating the arbitrary constants 'a' and 'b' from $z=ax^2+by^2$.
7. Form the partial differential equation by eliminating the arbitrary function from $z^2-xy=f\left(\frac{x}{z}\right)$.
8. Form the partial differential equation by eliminating the arbitrary function from $z=f\left(\frac{y}{x}\right)$.
9. Form the PDE by eliminating the arbitrary function from $\phi\left(x^2-y^2,z\right)=0$.
10. Find the complete integral of $p+q=pq$.
11. Find the complete integral of $p+q=1$.
12. Solve the partial differential equation $pq=x$.
13. Solve the equation $\left(D^4-D'^4\right)z=0$.
14. Solve $\left(D^2-7DD'+6D'^2\right)z=0$
15. Find the particular integral of $\left(D^2-2DD'+D'^2\right)z=e^{x-y}$.
16. Solve $(D-1)(D-D'+1)z=0$.

UNIT-II
FOURIER SERIES

PART-B

1. Express $f(x)=(\pi-x)^2$ as a Fourier Series of period 2π in the interval $0<x<2\pi$. Hence Deduce the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....$ (2012)
2. Find the Fourier series of $f(x)=\begin{cases}1 & for\ 0<x<\pi\\ 2 & for\ \pi<x<2\pi\end{cases}$ (2013)
3. Find the Fourier Series of for $f(x)=\begin{cases}x & (0,\pi)\\ 2\pi-x & (\pi,2\pi)\end{cases}$ and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$. (2010)

4. Expand in Fourier series of $f(x)=x\sin x$ for $0<x<2\pi$ and deduce that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-.....=\frac{\pi-2}{4}$. (2008,2014)
5. Find the Fourier expansion of $f(x)=x$ in $-\pi<x<\pi$. (2014)
6. Find the Fourier expansion of $f(x)=x^2$ in $-\pi<x<\pi$ and deduce that i). $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....=\frac{\pi^2}{6}$.
   ii). $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-.....=\frac{\pi^2}{12}$. iii). $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$ (2014 R13)
7. Find the Fourier series of $f(x)=x+x^2$ in $(-\pi,\pi)$ of periodicity 2π. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$. (2012)
8. Obtain the Fourier series to represent the function $f(x)=|x|$ in $-\pi<x<\pi$ and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$. (2012)
9. Find the Fourier series representation for $f(x)=|\sin x|$ in $-\pi<x<\pi$. (2015 R13)
10. Express $f(x)=x\sin x$ as a Fourier series in $(-\pi,\pi)$. (2011)
11. Find the F.S of $f(x)=\begin{cases}1+\frac{2x}{\pi}, & -\pi<x<0\\ 1-\frac{2x}{\pi}, & 0<x<\pi\end{cases}$. Hence, deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$ (2014)
12. Find the Fourier series expansion of $f(x)=\begin{cases}-x+1 & -\pi<x<0\\ x+1, & 0<x<\pi\end{cases}$. (2011, 2013)
13. Find the Fourier series of the function $f(x)=2x-x^2$ for $0<x<3$ and $f(x+3)=f(x)$. (2011)
14. Obtain Fourier series for $f(x)$ of period 2l and defined as follows $f(x)=\begin{cases}l-x, & 0<x\le l\\ 0 & l\le x\le 2l\end{cases}$.
15. Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-...=\frac{\pi}{4}$ and $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$. (2007)
16. Find the Fourier series expansion of period 2l for for the function $f(x)=(l-x)^2$ in the range $(0,2l)$.
    Deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.
17. Expand $f(x)=e^{-x}$ as Fourier series in $(-1,1)$. (2007,2008)
18. Find the F.S of $f(x)=x^2+x$ in $(-2,2)$. Hence find the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.....\infty$.
19. Find the Half range Fourier sine series for $f(x)=x$ in $(0,l)$. (2008)
20. If $f(x)=lx-x^2$ in the range $(0,l)$, show that the half range sine series for
    $f(x)=\frac{8l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin\left[(2n+1)\frac{\pi x}{l}\right]$. (2013)

21. Find the Half-range sine series of $f(x)=4x-x^2$ in the interval $(0,4)$. Hence, deduce the value of the series $\frac{1}{1^2}-\frac{1}{3^2}+\frac{1}{5^2}-.....\infty$. (2014)

22. Find Half range sine series of $f(x)=1$ in $(0,2)$. (2013)

23. Find the Half-range cosine series for $f(x)=(x-1)^2$ in $(0,1)$. Hence show that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.(2013,2014).

24. Find the half range cosine series of $f(x)=x$ in $0<x<\pi$. (2007,2010, 2012)

25. Find the half range cosine series of $f(x)=x\sin x$ in $0<x<\pi$. (2011)

26. Find the Half-range cosine series for the function $f(x)=x(\pi-x)$ in $0<x<\pi$. (2007)

27. Find the Fourier series of x^2 in $(-\pi,\pi)$. Use Parseval's identity to prove $\frac{\pi^4}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+.....$(2015 R13)

28. Find the cosine series for $f(x)=x$ in $(0,\pi)$ and then using Parseval's theorem. Show that $\frac{\pi^4}{96}=1+\frac{1}{3^4}+\frac{1}{5^4}+......$ (2007)(2014)

29. Find the Half-range cosine series for the function $f(x)=x(\pi-x)$ in $0<x<\pi$. Deduce that $\frac{\pi^4}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+.....$ (2007)

30. Find the Half range cosine series of $f(x)=(\pi-x^2)$ in the interval $(0,\pi)$. Hence find the sum of the series $1+\frac{1}{2^4}+\frac{1}{3^4}+.....+\infty$

31. Find the Fourier series upto the third harmonic for $y=f(x)$ in $(0,2\pi)$ defined by the table of values given below (2013, 2014, 2014R13,2015R13)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1.0

32. The values of x and the corresponding values of $f(x)$ over a perios T are given below, Show that $f(x)=0.75+0.37\cos\theta+1.004\sin\theta$ where $\theta=\frac{2\pi x}{T}$. (2009, 2011, 2015R13)

x	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
f(x)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

33. Find the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of y as given in the following table. (2006, 2010, M/J20012, N/D 2012)

x	0	1	2	3	4	5	6
f(x)	9	18	24	28	26	20	9

34. Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1\le x\le 1$. (2009) (2015 R13)

35. Find the complex form of the Fourier series of $f(x)=e^{x}$ in $-\pi\le x\le \pi$. (2009) (2015 R13)

36. Find the complex form of the Fourier series of the periodic function $f(x)=\sin x, 0<x<\pi$. (2014)
37. Find the complex form of the Fourier series of $f(x)=\cos ax, -\pi<x<\pi$,where 'a' is not an integer.(2013)

TWO MARKS

1. Write down the Dirichlet's conditions for a function to be expanded as a Fourier series. (or) State the conditions for f(x) to have Fourier series expansion.(or) Explain Ditichlet's conditions.
2. Find the constant term in the expansion of ???? ? as a Fourier series in the interval (-π,π).
3. Write a_0, a_n in the expansion of $x+x^3$ as a Fourier series in $(-\pi,\pi)$.
4. Find the Fourier constant ?? ??? ???? ?? (−?, ?).
5. Determine the Fourier series for the function $f(x)=x$ in $-\pi\le x\le\pi$.
6. Give the expression for the Fourier series coefficient b_n for the function $f(x)$ defined in $(-2,2)$.
7. What are the constant term a_0 and the coefficient of $\cos nx$, a_n in the Fourier series expansion of $f(x)=x-x^3$ in $(-\pi,\pi)$.
8. In the Fourier expansion of ?(?) = ? {1 + ??/? ?? − ? < ? < 0 ; 1 − ??/? ?? 0 < ? < ?} ?? (−?, ?). Find the value of ??, the coefficient of sin nx.
9. The cosine series for $f(x)=x\sin x$ for $0<x<\pi$ is given as $x\sin x=1-\frac{1}{2}\cos x-2\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2-1}\cos nx$. Deduce that $1+2\left[\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-....\right]=\frac{\pi}{2}$.
10. If $(\pi-x)^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ in $0\le x\le 2\pi$ then deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.
11. Write down parsevals formulas on Fourier coefficients.
12. If $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$ then deduce the value of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+......=\frac{\pi^2}{6}$.
13. Find the value of a_0 in the Fourier series expansion of $f(x)=e^x$ in $(0,2\pi)$.
14. Find a Fourier sine series for the function ?(?) = 1 ?? 0 < ? < ?
15. Define root mean square value of a function $f(x)$ in a<x<b.
16. Find the root mean square value of the function $f(x)=x$ in the interval $(0,l)$.
17. If $f(x)=x^2+x$ is expressed as a Fourier series in the interval $(-2,2)$ to which value this series converges at x=2?
18. If f(x) is discontinuous at x=a .What does its Fourier series represent at that point.
19. Without finding the values of ??, ?? ??? ?? the Fourier coefficients of Fourier series for the function ?(?) = ?? in the interval (0,π). Find the value of ???/2 + ∑ (???? + ???) (sum over ???, upper limit ?)

UNIT-III

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART-B

1. A string is stretched and fastened to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string into the form $y=k\left(lx-x^2\right)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time t.
2. A string of length $2l$ is fastened at both ends. The midpoint of the string is taken to a height b and then released from rest in that position. Find the displacement of the string.
3. A tightly stretched string with fixed end points $x=0$ and $x=l$ initially displaced in a sinusoidal arc of length y_0 and then released from rest. Find the displacement y at any distance x from one end at time t. (2014)
4. A uniform elastic string of length 60cm is subjected to a constant tension of 2kg. If the ends are fixed and the initial displacement is $y(x,0)=60x-x^2$ for $0<x<60$ while the initial velocity is zero, find the displacement function $y(x,t)$.
(2015R8)
5. A string is tightly stretched and its ends are fastened at two points $x=0$ and $x=l$. The midpoint of the string is displaced transversely through a small distance b and the string is released from rest in that position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.
6. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity $\lambda x(l-x)$. Find the displacement of the string.
7. If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $v_0 \sin^3 \frac{\pi x}{l}, 0<x<l$, determine the displacement of a point distant x from one end at time 't'.
8. A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocity $v=\begin{cases} \frac{cx}{l} & if\ 0\le x\le l \\ \frac{c}{l}(2l-x) & if\ l\le x\le l \end{cases}$. Find the displacement of the string at any subsequent time.
9. Solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ subject to the conditions

$i)\ u(0,t)=0 \quad \forall t>0$

$ii)\ u(l,t)=0 \quad \forall t>0$ (2013, 2015)

$iii)\ u(x,0)=\begin{cases} x, & 0\le x\le l/2 \\ l-x, & l/2\le x\le l \end{cases}$

10. . A rod of l long has its ends A and B kept at 20° and 80° respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^\circ C$ and kept so. Find the resulting temperature function $u(x,t)$ taking x=0 at A.

11. A rod 30cm long has its ends A and B kept at $20^\circ C$ and $80^\circ C$ respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^\circ C$ kept so. Find the resulting temperature function $u(x,t)$ taking $x=0$ at A.
12. The ends A and B of a rod l cm long have their temperature kept at $30^\circ C$ and $80^\circ C$, until steady state conditions prevail. The temperature of the end B is suddenly reduced to $60^\circ C$ and that of A is increased to $40^\circ C$. Find the temperature distribution in the rod after time t.
13. The ends A and B of a rod 40cm long have their temperature kept at $0^\circ C$ and $80^\circ C$, until steady state conditions prevail. The temperature of the end B is suddenly reduced to $40^\circ C$ and kept so, while that of the end A is kept at $0^\circ C$. Find the temperature distribution in the rod after time t.
14. A rod, 30cm long , with indulated sides, has its ends A and B kept at $20^\circ C$ and $80^\circ C$ respectively until steady state conditions prevail. The temperature at A is then suddenly raised to $40^\circ C$ and at the same time that at B is lowered to $60^\circ C$. Find the subsequent temperature at any point of the rod at any time.
15. An infinitely long rectangular plate with insulated surfaces is 10cm wide. The long two edges and one short edge are kept at $0^\circ C$, while the other short edge $x=0$ is kept at temperature
$u=\begin{cases}20y, & 0\le y\le 5\\ 20(10-y), & 5\le y\le 10\end{cases}$. Find the temperature distribution in the plate.
16. An infinitely long rectangular plate with insulated surfaces is 10cm wide. The long two edges and one short edge are kept at $0^\circ C$, while the other short edge $y=0$ is kept at temperature
$u=\begin{cases}20x, & 0\le x\le 5\\ 20(10-x), & 5\le x\le 10\end{cases}$. Find the steady state temperature at point on it.

TWO MARKS

1. A string is stretched and fastened to two points / apart. Motion is started by displacing the string into the form ? = ?? sin ??/? from which it is released at time t = 0. Formulate this problem as the boundary value problem.
2. What is the constant ?? in the wave equation ??? = ?? ??? ?

   (OR)

   In the wave equation ???/??? = c? ???/??? what does ?? stand for?
3. State the suitable solution of the one dimensional heat equation ??/?? = a? ???/???
4. State the governing equation for one dimensional heat equation and necessary conditions to solve the problem.
5. Write all variable separable solutions of the one dimensional heat equation ?? =∝? ??? .
6. Write down the diffusion problem in one-dimensional as a boundary value problem in two different forms.
7. State any two laws which are assumed to derive one dimensional heat equation.
8. Write any two solutions of the Laplace equation ??? + ??? = 0 involving exponential terms in ? or ?.
9. In steady state conditions derive the solution of one dimensional heat flow equation.

10. Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement f(x) and initial velocity g(x).
11. Write down the governing equation of two dimensional steady state heat conduction
12. The ends A and B of a rod of length 10 cm long have their temperature kept at 20°C and 70°C. Find the steady state temperature distribution on the rod.
13. Write down the three possible solutions of Laplace equation in two-dimensions.

UNIT-IV
FOURIER TRANSFORM

PART-B

1. Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & |x|<a \\ 0, & |x|>a \end{cases}$ and Hence, deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}$ and $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt=\frac{\pi}{15}$. (2008,2009,2011,2012)
2. Find the Fourier transform of $f(x)=\begin{cases} 1-x^2, & |x|<1 \\ 0, & |x|>1 \end{cases}$ and Hence, deduce that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}ds=\frac{3\pi}{16}$ and $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds=\frac{\pi}{15}$. (2011,2013)
3. Find the Fourier Transform of $f(x)=\begin{cases} a-|x|, & |x|<a \\ 0, & |x|>a>0 \end{cases}$. Hence deduce that i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$, ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$. (2015 R13)
4. Find the Fourier Transform of $f(x)=\begin{cases} 1-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$.Hence deduce that i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$, ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$. (2009,2011,2012,2014R13)
5. Find the Fourier transform of $f(x)=\begin{cases} 1, & |x|<a \\ 0, & |x|>a \end{cases}$, where 'a' is a Positive real number. Hence deduce that i). $\int_0^\infty \frac{\sin t}{t}dt=\frac{\pi}{2}$ ii). $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$. (2008,2011,2013,2015R13)
6. Show that the Fourier transform of $f(x)=e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$ (or) Show that $e^{-\frac{x^2}{2}}$ is self-reciprocal with respect to Fourier Transform. (2011,2013)
7. Find the Fourier transform of $e^{-a^2x^2}$, a>0, Hence show that $e^{-\frac{x^2}{2}}$ is self reciprocal under Fourier transform. (2014 R8&R13,2015)
8. Find Fourier transform of $e^{-a|x|}$ and hence deduce that i) $F\left[xe^{-a|x|}\right]=i\sqrt{\frac{2}{\pi}}\frac{2as}{(s^2+a^2)}$ ii) $\int_0^\infty \frac{\cos xt}{a^2+t^2}dt=\frac{\pi}{2a}e^{-a|x|}$. (2012,2014,2014R8,2015R8)
9. Find Fourier transform of $e^{-a|x|}$ and hence deduce that i) $F\left[xe^{-a|x|}\right]=i\sqrt{\frac{2}{\pi}}\frac{2as}{(s^2+a^2)}$ ii) $\int_0^\infty \frac{\cos xt}{a^2+t^2}dt=\frac{\pi}{2a}e^{-a|x|}$.

(2012,2014,2014R8,2015R8)

10. State and prove Convolution theorem. (2011,2012)
11. Verify convolution theorem for $f(x)=g(x)=e^{-x^2}$. (2013)
12. Find the Fourier sine and cosine transform of $f(x)=e^{-\alpha x},\alpha>0$ and hence deduce the inversion formula. (2012,2014 R8)
13. Show that $e^{-\frac{x^2}{2}}$ is self-reciprocal under Fourier cosine transform. (2007)
14. Find the Fourier cosine transform of $e^{-a^2x^2}$. (2012)
15. Evaluate $F_C\left[x^{n-1}\right]$ if $0<x<1$ and Hence show that $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier cosine transform.
16. Find the Fourier sine transform of $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$
17. Find the Fourier sine transform of the function $f(x)=\frac{e^{-ax}}{x}$ and hence find $F_S\left[\frac{e^{-ax}-e^{-bx}}{x}\right]$.
18. Find $f(x)$ if its sine transform is $\frac{e^{-sa}}{s}$.Hence find $F_S^{-1}\left[\frac{1}{s}\right]$.
19. Find the Fourier sine transform of e^{-ax} and hence find the Fourier cosine transform of xe^{-ax}.
20. Solve the integral equation $\int_0^\infty f(x)\cos\lambda x\,dx=e^{-\lambda}$ and also show that $\int_0^\infty \frac{\cos\lambda x}{1+x^2}dx=\frac{\pi}{2}e^{-\lambda}$ (2015 R8)
21. Solve the integral equation $\int_0^\infty f(x)\sin sx\,dx=\begin{cases} 1, & 0\le s<1 \\ 2, & 1\le s<2 \\ 0, & s\ge 2 \end{cases}$. (2014)\
22. Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using transforms. (2014R13)
23. Evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ using transforms. (2010)
24. Using transform methods, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ (2013,2014)
25. Using transform methods, evaluate $\int_0^\infty \frac{x^2dx}{(x^2+a^2)^2}$ (2014)

TWO MARKS

1. State Fourier integral theorem.
2. Show that $f(x)=1,\ 0<x<\infty$ cannot be represented by a Fourier integral. (2014)
3. Define Fourier Transform pair. (2010,2011)
4. Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$. (2014)
5. Define self reciprocal with respect to Fourier transform (2013)
6. Find the Fourier transform of $f(x)=\begin{cases} e^{ikx}, & a<x<b \\ 0, & x<a \text{ and } x>b \end{cases}$ (2014)

7. prove the shifting property on Fourier transform of $f(x)$

(OR) (2012,2013)

What is the Fourier transform of $f(x-a)$ if the Fourier transform of $f(x)$ is $F(s)$

8. If $F(s)$ is the Fourier transform of $f(x)$, then $F[f(x)\cos ax]=\frac{1}{2}[F(s+a)+F(s-a)]$.

(or) State and prove Modulation property. (2015R13)

9. State and prove the change of scale property of Fourier transform (or) If $F[f(x)]=F(s)$ then $F[f(ax)]=\frac{1}{a}F\left[\frac{s}{a}\right]$.

10. prove that $F_S[xf(x)]=-\frac{d}{ds}[F_C(s)]$. (2015 R8)

11. prove that $F_C[xf(x)]=\frac{d}{ds}[F_S(s)]$. (2015 R8)

12. Find the Fourier cosine transform of e^{-ax} , a>0. (2014R8)

13. Find the Fourier cosine transform of e^{-ax} , a>0. (2014R8)

14. Find the Fourier cosine transform of e^{-3x} . (2013)

15. Find the Fourier sine transform of $\frac{1}{x}$.

16. State the convolution theorem for Fourier Transform.

UNIT-V
Z-TRANSFORMS AND DIFFERENCE EQUATIONS

PART-B

1. Find the Z-transform of $\cos n\theta$ and $\sin n\theta$. (2010)
2. Find the Z-transforms of $a^n \cos n\theta$ and hence deduce $Z\left(\cos\frac{n\pi}{2}\right)$ (2011)
3. Find the Z-transforms of $a^n \cos n\theta$ and $e^{-at}\cos bt$. (2014)
4. Find the Z-transform of $\frac{1}{(n+1)(n+2)}$. (2013,2014)
5. Find the Z-transform of $\frac{1}{n(n+1)}, n\geq 1$. (2014)
6. Find $Z(r^n \sin n\theta)$. (2015)
7. Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ and $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$. (2009)
8. Find the inverse Z-transform of $\frac{10z}{z^2-3z+2}$. (2009)
9. Find the inverse Z-transform of $\frac{z^3-20z}{(z-2)^3(z-4)}$.(2009)

10. Find the inverse Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$, using partial fraction.(2014)
11. Find the inverse Z-transform of $\frac{z(z+1)}{(z-1)^3}$ by residue method. (2010)
12. Using residue method, find $Z^{-1}\left[\frac{z}{z^2-2z+2}\right]$. (2014)
13. Using complex residue theorem, Evaluate $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$. (2015)
14. Using Convolution theorem, find the inverse Z-transform of $\left(\frac{z}{z-4}\right)^3$. (2009)
15. State and prove convolution theorem. (2012)
16. Using convolution theorem, find the Z^{-1} of $\frac{z^2}{(z-4)(z-3)}$. (2010)
17. Using convolution theorem, find the Z^{-1} of $\frac{z^2}{(z-1)(z-3)}$. (2011, 2013)
18. Using Convolution theorem, find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$ (or) $\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$.(2012,2014,2015)
19. Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z+a)^2}$.(2012)
20. Using Convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$. (2013,2014,2015)
21. Form the difference equation from the relation $y_n = a+b.3^n$. (2010)
22. Derive the difference equation from $y_n = (A+Bn)(-3)^n$.(2011)
23. Form the difference equation of second order by eliminating the arbitrary constants A and B from $y_n = A(-2)^n + Bn$. (2011)
24. Form the difference equation from $y(n) = (A+Bn)(2)^n$. (2013)
25. Solve by Z-transform $u_{n+2}-2u_{n+1}+u_n = 2^n$ with $u_0 = 2$ *and* $u_1 = 1$. (2009)
26. Solve by Z-transform $u_{n+2}+4u_{n+1}+3u_n = 2^n$ with $u_0 = 0$ *and* $u_1 = 1$ using Z-transform. (2009)
27. Solve the equation $u_{n+2}+6u_{n+1}+9u_n = 2^n$ given $u_0 = u_1 = 0$. (2009 ,2012)
28. Solve the difference equation $y(n+3)-3y(n+1)+2y(n) = 0$, given $y(0) = 4, y(1) = 0$ *and* $y(2) = 8$ (2011,2012,2014)
29. Solve $u_{n+2}+4u_{n+1}+3u_n = 3^n$ given that $u_0 = 0$ *and* $u_1 = 1$. (2011,2015)
30. Solve $y_{n+2}-3y_{n+1}-10y_n = 0$, $y_0 = 1$ *and* $y_1 = 0$. (2013,2014)
31. Using Z-transform solve the difference equation $y_{n+2}+2y_{n+1}+y_n = n$ given $y_0 = 0 = y_1$.
32. Solve by Z-transform $u_{n+2}-3u_{n+1}+2u_n = 4^n$ with $u_0 = 0$ *and* $u_1 = 1$ using Z-transform. (2014,2014R13)
33. Using Z-transform, solve the difference equation $x(n+2)-3x(n+1)+2x(n) = 0$ given $x(0) = 0, x(1) = 1$. (2015)

TWO MARKS

1. Find the Z-transform of a^n.
2. Find $Z(n)$.
3. Find the Z-transform of $\frac{1}{n}$.
4. Find the Z-transform of n^2.
5. Find the Z-transform of $x(n)=\begin{cases} \frac{a^n}{n!} & for\ n\geq 0 \\ 0 & otherwise \end{cases}$.
6. Prove that $Z\left[a^n f(n)\right]=\bar{f}\left(\frac{z}{a}\right)$. (2014,2015)
7. If $Z\left[f(n)\right]=\bar{f}(z)$ then prove that $Z\left[f(-n)\right]=\bar{f}\left(\frac{1}{z}\right)$.
8. Define the unit step sequence. Write its Z-transform.
9. Obtain $Z^{-1}\left(\frac{z}{(z+1)(z+2)}\right)$.
10. Find the inverse Z transform of $\frac{z}{(z+1)^2}$.
11. Form a difference equation by eliminating the arbitrary constant A form $y_n = A3^n$.
12. Form a difference equation by eliminating the arbitrary constant A form $u_n = A2^{n+1}$.
13. What advantage is gained when Z-transform is used to solve difference equation?
14. Solve $y_{n+1}-2y_n=0$ given $y_0=3$
15. Solve $y_{n+1}-2y_n=0$ given $y_0=2$
16. State initial value and Final value theorem on Z-transforms.
17. State the convolution theorem on Z-transfroms.